CHIRAL SYMMETRY IN LINEAR SIGMA MODEL IN MAGNETIC ENVIRONMENT

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Abstract

We study the chiral symmetry structure in a linear sigma model with fermions in the presence of an external, uniform magnetic field in the 'effective potential' approach at the one loop level. We also study the chiral phase transition as a function of density in the core of magnetized neutron stars.

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In the absence of quark mass matrix, QCD is invariant under chiral transformation at the lagrangian level. However, the dynamics of QCD are expected to be such that chiral symmetry is spontaneously broken with the vacuum state having a nonzero quark-antiquark condensate $\langle 0 \left| \bar{q}q \right| 0 \rangle$ and the Goldstone theorem then requires the existence of approximately massless pseudo-scalar mesons in the hadron spectrum. At high temperatures and/or at high densities,the quark condensates are expected to melt at some critical point and chiral symmetry is restored [1]. Chiral phase transition and phenomenological consequences in the form of experimentally observable signatures have been extensively discussed in the literature [2]. It has also been suggested[3, 4] that systems with spontaneously broken symmetries may make a transition from broken symmetric to restored symmetric phase in the presence of external fields. It has been shown [5, 6] that there exist some realistic models where the symmetry restoration takes place. In QED uniform, external static magnetic field is known to break chiral symmetry dynamically at weak gauge couplings [7].

Large magnetic fields with a strength of 10^{18} gauss or even more, have been conceived to exist at the time of supernova collapse inside neutron stars and in other astrophysical compact objects and in the early Universe. Effect of such a strong magnetic field on chiral phase transition is thus of great interest for baryon free quark matter in the early universe and for high density baryon matter in the core of neutron stars. To study chiral phase transition in QCD we need a nonperturbative treatment. Lattice techniques and the Schwinger-Dyson equations provide specially powerful methods to study the chiral structure of QCD [9]. A particularly attractive frame work to study such systems is the linear sigma model originally proposed as a model for strong nuclear interactions [10]. We will consider this as an effective model for low energy phase of QCD. To fix ideas we consider a two flavor $SU(2) \times SU(2)$ chiral quark model given by the lagrangian.

$$\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}.\vec{\pi})\psi + \frac{1}{2}(\partial_{\mu}\sigma)^2 + \frac{1}{2}(\partial_{\mu}\vec{\pi})^2 - U(\sigma, \vec{\pi}) \quad (1)$$

where ψ is the quark field σ and π are the set of four scalar fields and g is the quark meson coupling constant. The potential U$(\sigma, \vec{\pi})$ is given by

$$U(\sigma, \vec{\pi}) = -\frac{1}{2}\mu^2(\sigma^2 + \vec{\pi}^2) + \frac{1}{4}\lambda(\sigma^2 + \vec{\pi}^2)^2 \quad (2)$$

For $\mu^2 > 0$ chiral symmetry is spontaneously broken. The σ field can be used to represent the quark condensate, the order parameter for chiral phase transition and the pions are the Goldstone bosons. At the tree level the sigma , pion and the quark masses are given by

$$m_{\sigma}^2 = 3\lambda\sigma_{cl}^2 - \mu^2; m_{\pi}^2 = \lambda\sigma_{cl}^2 - \mu^2; m_{\psi}^2 = g\sigma_{cl} \quad (3)$$

where $\sigma_{cl}^2 = \frac{\mu^2}{\lambda} = f_{\pi}^2$. An elegant and efficient way to study symmetry properties of the vacuum at finite temperature, density and in the presence of external fields is through the "Effective Potential" approach discussed extensively in the literature [11]. We will compute here, in the one loop approximation, the effective potential in the presence of external magnetic field which is defined through an effective action $\Gamma(\sigma, B)$ which is the generating functional of the one particle irreducible graphs. The effective potential is then given by

$$V_{eff}(\sigma, B) = V_0(\sigma) + V_1(\sigma, B) \tag{4}$$

where $V_1(\sigma, B)$ is obtained from the propagator function $G(\sigma, B)$ by the usual relation $V_1(\sigma, B) = -\frac{1}{2i} T_r \log G(\sigma, B)$.

Alternatively one can compute the shift in the vacuum energy density due to zero-point oscillations of the fields considered as an ensemble of harmonic oscillators [11]. We thus require energy eigenvalues(excitations) of particles in the magnetic field, which can be easily obtained, and in the absence of anomalous magnetic moment for uniform static magnetic field in the z-direction for a particle of mass M,charge q and spin J, are given by [12]

$$E(k_z, n, J_z) = \left(k_z^2 + M^2 + (2n + 1 - sign(q)\ j_z) \left|q\right| B\right)^2 \tag{5}$$

where n represents the landau level. Contribution of scalar particles of mass M to $V_1(M^2)$ after wick rotation is thus given by

$$V_1(M^2) = \frac{1}{2} \int \frac{d^4 k_e}{(2\pi)^4} ln(k_e^2 + M^2 - i\epsilon) \tag{6}$$

In the presence of magnetic field, all we need to do is to replace the phase space integral $\int \frac{d^4 k_e}{(2\pi)^4}$ by $\frac{eB}{2\pi} \sum_{n=0}^{\infty} \frac{d^2 k_e}{(2\pi)^2}$ and the energy by expression (5) for charged particles. For a scalar field of charge $\pm e$, we thus have

$$
\begin{aligned}
V_1(M^2,B) &= \frac{eB}{4\pi}\sum_{n=0}^{\infty}\int\frac{d^2k_e}{(2\pi)^2}\; ln(k_e^2+(2n+1)eB+M^2) \\
&= -\frac{eB}{4\pi}\;\frac{\partial}{\partial\alpha}\;\frac{\Gamma(\alpha-d/2)}{\Gamma(\alpha)(4\pi)^{d/2}} \\
&\quad \sum_{n=0}^{\infty}\frac{1}{(M^2+eB+2neB)^{\alpha-d/2}}\Big|_{\alpha=0,d=2} \\
&= -\frac{eB}{4\pi}\;\lim_{\alpha\to 0}\;\frac{\partial}{\partial_\alpha}\;\frac{\Gamma(\alpha-\frac{d}{2})}{\Gamma(\alpha)(4\pi)^{\alpha-\frac{d}{2}}} \\
&\quad \frac{1}{(2eB)^{\alpha-\frac{d}{2}}}\;\zeta(\alpha-\frac{d}{2},\frac{M^2+eB}{2eB})\Big|_{\alpha=0,d=2} \qquad (7)
\end{aligned}
$$

where $\zeta(z,q)$ is the generalized Riemann zeta function

$$
\zeta(z,q)=\sum_{n=0}^{\infty}\frac{1}{(q+n)^2}=\frac{1}{\Gamma(z)}\int_0^{\infty}dt\;\frac{t^{z-1}e^{-qt}}{1-e^-t} \qquad (8)
$$

The potential (7) has poles at α=0, 1 and 2 for d=2 which can be absorbed in the counter terms. The finite part depends on the exact renormalization conditions that are imposed. In what follows we would use the $\overline{MS}$ renormalization scheme. From eqns. (7) and (8) we can write

$$
V_1(M^2,B)=-\frac{eB}{32\pi^2}\lim_{\alpha\to 0}\frac{\partial}{\partial\alpha}\frac{(2eB)^{1-\alpha}}{\Gamma(\alpha)}\int dt\; t^{\alpha-2}\;\frac{e^{-\frac{M^2}{2eB}t}}{\sinh\frac{t}{2}} \qquad (9)
$$

which converges for Re $\alpha > 2$. We analytically continue the result in the complex α-plane and use dimensional regularization technique to extract the finite contribution. To proceed further we first consider the case $\frac{M^2}{2eB} > 1$, expand $e^{-\frac{M^2}{2eB}t}$ and formally integrate (9) to obtain [13]

$$V_1(M^2, B) = -\frac{|q|\, B}{32\pi^2} \lim_{\alpha\to 0} \frac{\partial}{\partial\alpha} \sum_{n=0}^{\infty} (\frac{M^2}{2eB})^{\alpha+\nu-1} \frac{(-1)^\nu}{\nu!\; (M^2)^{\alpha-1}} \frac{2}{\Gamma(\alpha)}(2^{\alpha+\nu-1} - 1)\; \Gamma(\alpha+\nu-1)\; \zeta(\alpha+\nu-1) \quad (10)$$

Keeping leading terms in $\frac{M^2}{2eB}$ we obtain

$$V_1(M^2, B) = -\frac{1}{16\pi^2} [\; e^2B^2\, \frac{\pi}{2}\, \zeta(2) \log 2eB + eBM^2\frac{\pi^2}{2}\, \log 2 - M^4\frac{\pi}{2}\, \log 2eB + ..] \quad (11)$$

The leading term for the contribution of charged Goldstone bosons relevant for symmetry considerations is

$$V_1(M^2, B) \sim -\frac{eBM^2}{32} \log 2 \quad (12)$$

which agrees with Linde's result [6] upto a factor of order one. For the case of $\frac{2eB}{M^2} > 1$ we write (9) as

$$V_1(M^2, B) = -\frac{1}{32\pi^2} \lim_{\alpha\to 0} \frac{\partial}{\partial\alpha} \frac{1}{\Gamma(\alpha)} \int_0^\infty dx\; x^{\alpha-3} e^{-M^2x} \frac{eBx}{\sinh eBx} \quad (13)$$

and keeping leading terms obtain

$$V_1(M^2, B) \simeq \frac{1}{64\pi^2}[M^4(\log M^2 - \frac{3}{2}) - \frac{2}{3}(eB)^2 \log M^2] \quad (14)$$

which agrees with the result given by Salam and Strathdee [5]. Likewise for the charged fermion fields using (5) we obtain

$$V_1(M^2,B) = \frac{4\,|q|\,B}{32\pi^2}\lim_{\alpha\to 0}\frac{\partial}{\partial_\alpha}\frac{(2\,|q|\,B)^{1-\alpha}}{\Gamma(\alpha)}\int_0^\infty dt\ t^{\alpha-2}e^{-\frac{M^2}{2|q|B}t}\coth\frac{t}{2} \tag{15}$$

The factor of 4 and positive sign account for the spinor nature of the fermi field. In the limits mentioned above, we obtain

$$V_1(M^2,B) = \frac{|q|\,BM^2}{8\pi^2}(1-\log M^2) \tag{16}$$

and

$$V_1(M^2,B) \simeq -\frac{1}{16\pi^2}[M^4\ (\log M^2 - \frac{3}{2}) + \frac{2}{3}\ (|q|\,B)^2\log M^2] \tag{17}$$

for $\frac{2|q|B}{M^2} > 1$ and < 1 respectively.

The total $V_{eff}(\sigma,B)$ for the sigma model at the one loop level is thus given by

$$\begin{aligned}
V_{eff}(\sigma,B) &= -\frac{1}{2}\mu^2\sigma^2 + \frac{\lambda}{4}\sigma^2 \\
&+ \frac{1}{64\pi^2}\ (3\lambda\sigma^2-\mu^2)^2\ \log\ (\frac{3\lambda\sigma^2-\mu^2}{m_\sigma^2} - \frac{3}{2}) \\
&+ \frac{1}{64\pi^2}\ (\lambda\sigma^2-\mu^2)^2\ \log\ (\frac{\lambda\sigma^2-\mu^2}{m^2} - \frac{3}{2}) \\
&- \frac{eB}{16}\ (\lambda\sigma^2-\mu^2)\ \log 2 \\
&- \frac{N_c}{16\pi^2}\sum_{flav}[g^4\sigma^4\ (\ \log\frac{g^2\sigma^2}{m_f^2} - \frac{3}{2}) \\
&+ \frac{2}{3}\ (|q|\,B)^2\ \log\frac{g^2\sigma^2}{m_f^2}]
\end{aligned} \tag{18}$$

For $\frac{|q|B}{M^2} > 1$,the last term in eqn.(18) is replaced by eq. (7) summed over flavors. In figure 1, we plot $V_{eff}(\sigma, B)$ as a function of σ for different values of magnetic field and compare it with the case of zero magnetic field by ignoring the B independent one loop terms. As input parameters we choose the constituent quark mass to be 340 MeV, sigma mass m_σ=1 GeV and f_π=93 MeV. We find that in the presence of intense magnetic fields the chiral symmetry breaking is enhanced. For magnetic field large compared to m_f^2 , from eqn.(7) we observe that though the fermionic contribution is towards symmetry restoration, it is not enough to offset the contribution of charged goldstone pions. In order to study chiral symmetry restoration in the case of neutron stars as a function of chemical potential μ associated with finite baryon number density we employ the imaginary time formalism by summing over Matsubara frequencies. This amounts to adding the fermionic free energy to the one loop effective potential and is given by

$$V_1^\beta(\sigma) = -\frac{\gamma}{\beta}\int \frac{d^3k}{(2\pi)^3}\ \ln\,(1 + e^{-\beta(E-\mu)}) \tag{19}$$

which in the presence of static uniform magnetic field becomes

$$V_1^\beta(\sigma) = -\frac{\gamma}{\beta}\,\frac{eB}{2\pi}\sum_{n=0}^{\infty}\int_0^\infty \frac{dk_z}{2\pi}\ \ln\,(1 + e^{-\beta(E-\mu)}) \tag{20}$$

where γ is the degeneracy factor and is equal to $2N_c$ for each quark flavor. We consider cold dense isospin symmetric quark matter for which the integrals can be performed analytically. The baryon number density corresponding to the chemical potential μ is given by the usual thermodynamical relations.

$$N_B(\mu, 0) = \frac{1}{3} \sum_{flav} \frac{\gamma}{6\pi^2} \; (\mu^2 - g^2\sigma^2)^{\frac{3}{2}} \tag{21}$$

for B = 0 and

$$N_B(\mu, B) = \frac{1}{3} \sum_{n=0}^{n_{max}} \frac{\gamma\, |q|\, B}{4\pi^2} \; (2 - \delta_{\mu,0}) \; \sqrt{\mu^2 - g^2\sigma^2 - 2n\, |q|\, B} \tag{22}$$

where n_{max}= Int $\left|\frac{\mu^2 - g^2\sigma^2}{2|q|B}\right|$. To study chiral symmetry behavior at finite density in the presence of uniform magnetic field, we minimize effective potential with respect to the order parameter σ for fixed values of chemical potential and magnetic field (which then fixes the baryon density). The results are shown in figure 2 where we have plotted the order parameter σ as a function of density at T=0 for different values of magnetic field. The solution indicates a first order phase transition. The point where the curve cuts the density axis is not the point where chiral symmetry is restored. At the actual point of phase transition the two minima of the effective potential at $\sigma = 0$ and $\sigma(\mu, B)$ become degenerate. We find that magnetic field restores chiral symmetry at a lower density.

In conclusion we have examined the chiral symmetry behavior of the Linear Sigma model in the presence of static, uniform magnetic field at the one loop level at zero density and at densities relevant in the core of neutron stars. We find that the contribution of scalar and fermion loops leads to an increase in chiral symmetry breaking. However, at high densities the magnetic field enhances the restoration of chiral symmetry. This would have implications on the dynamics of neutron star evolution where such high densities and magnetic fields may be present. It is likely that in the core of neutron stars the nuclear matter may undergo a transition to deconfined quark matter. Existence of chirally broken quark phase would imply the presence of massive quark matter in the core of such stars. If the core is magnetized, the chiral symmetry will get restored at densities lower than if no magnetic field were present. This would affect the equation of state and will have astrophysical implications.

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Figure captions

Figure 1. Effective Potential in units of $(100MeV)^4$ as a function of $\sigma(MeV)$ for different values of the magnetic field. The solid, long dashed, dashed, dotted and dashed-dotted curves are for B=0, $10^2, 10^4, 10^5, 5 \times 10^5 MeV^2$ respectively.

Figure 2. Chiral condensate $\sigma(MeV)$ as a function of baryon density in f_m^{-3} for different values of the magnetic field. The curves are labeled as in fig 1.

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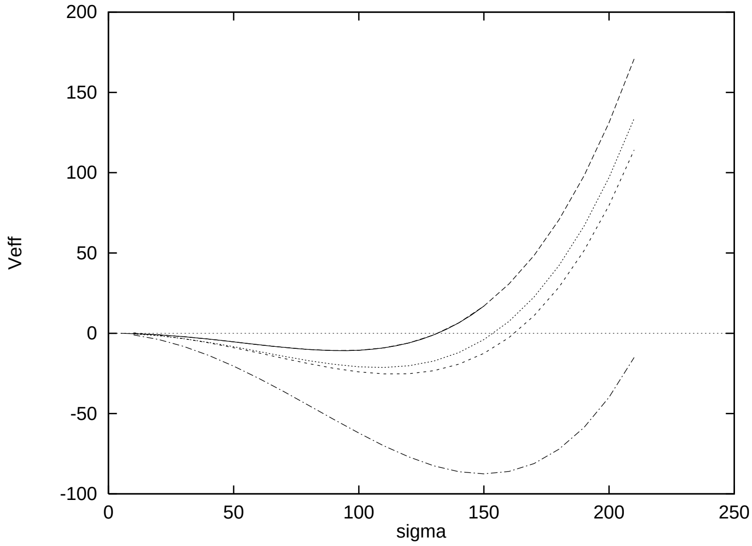

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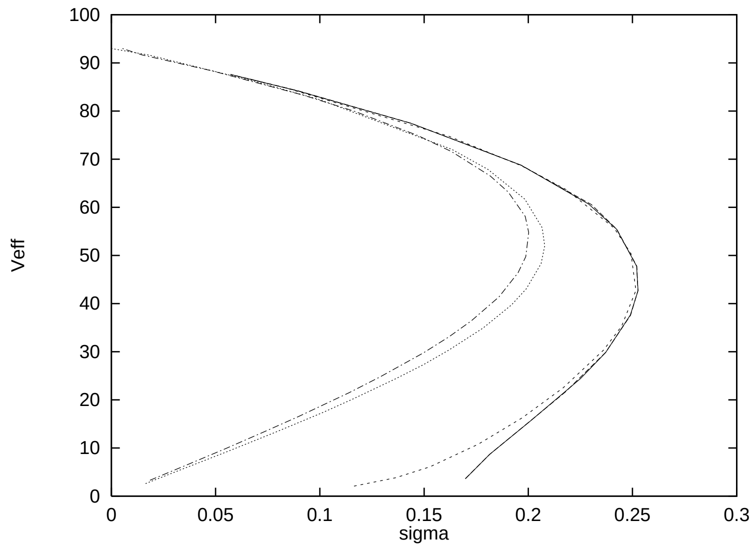

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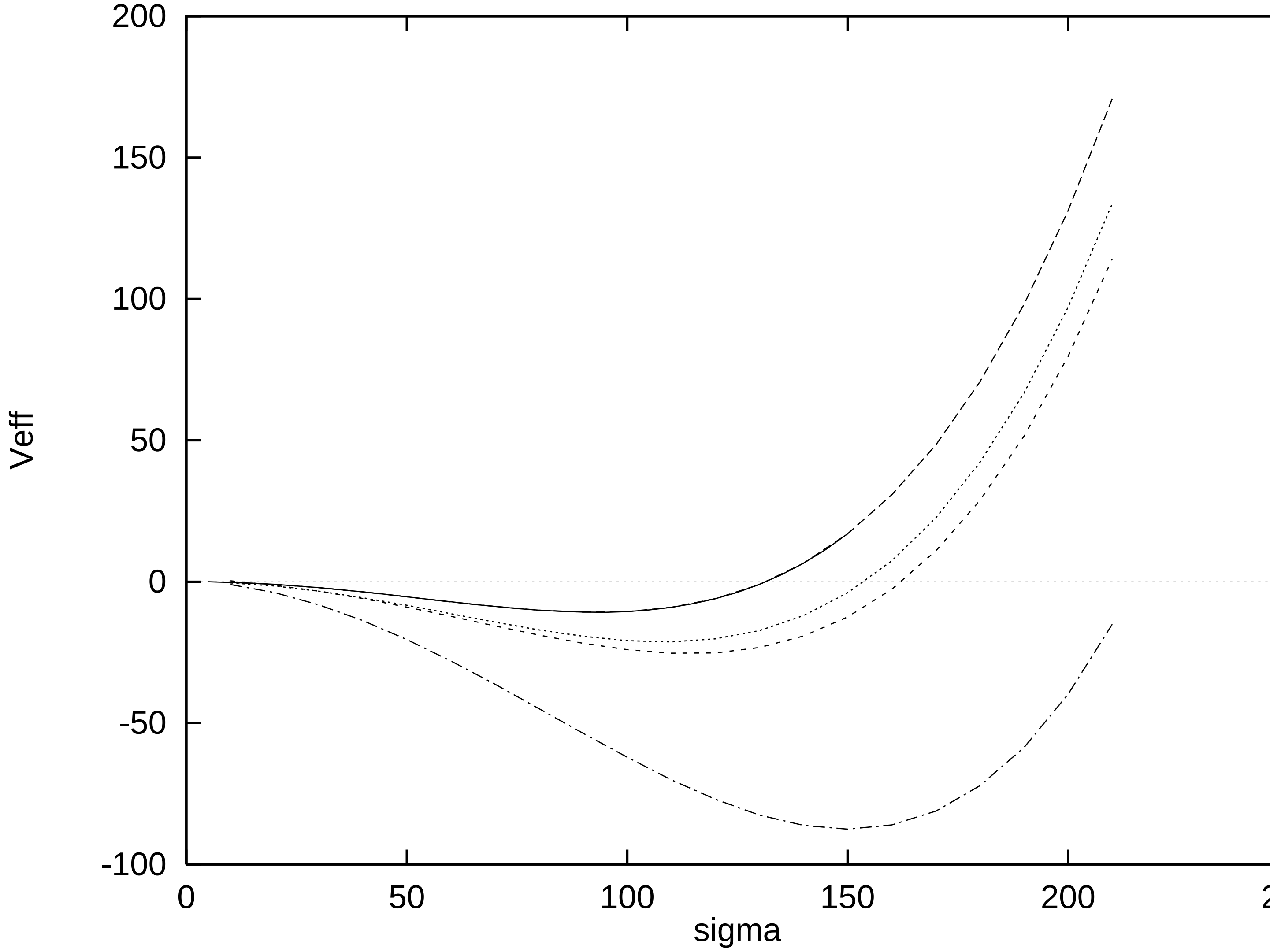
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-50
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Veff
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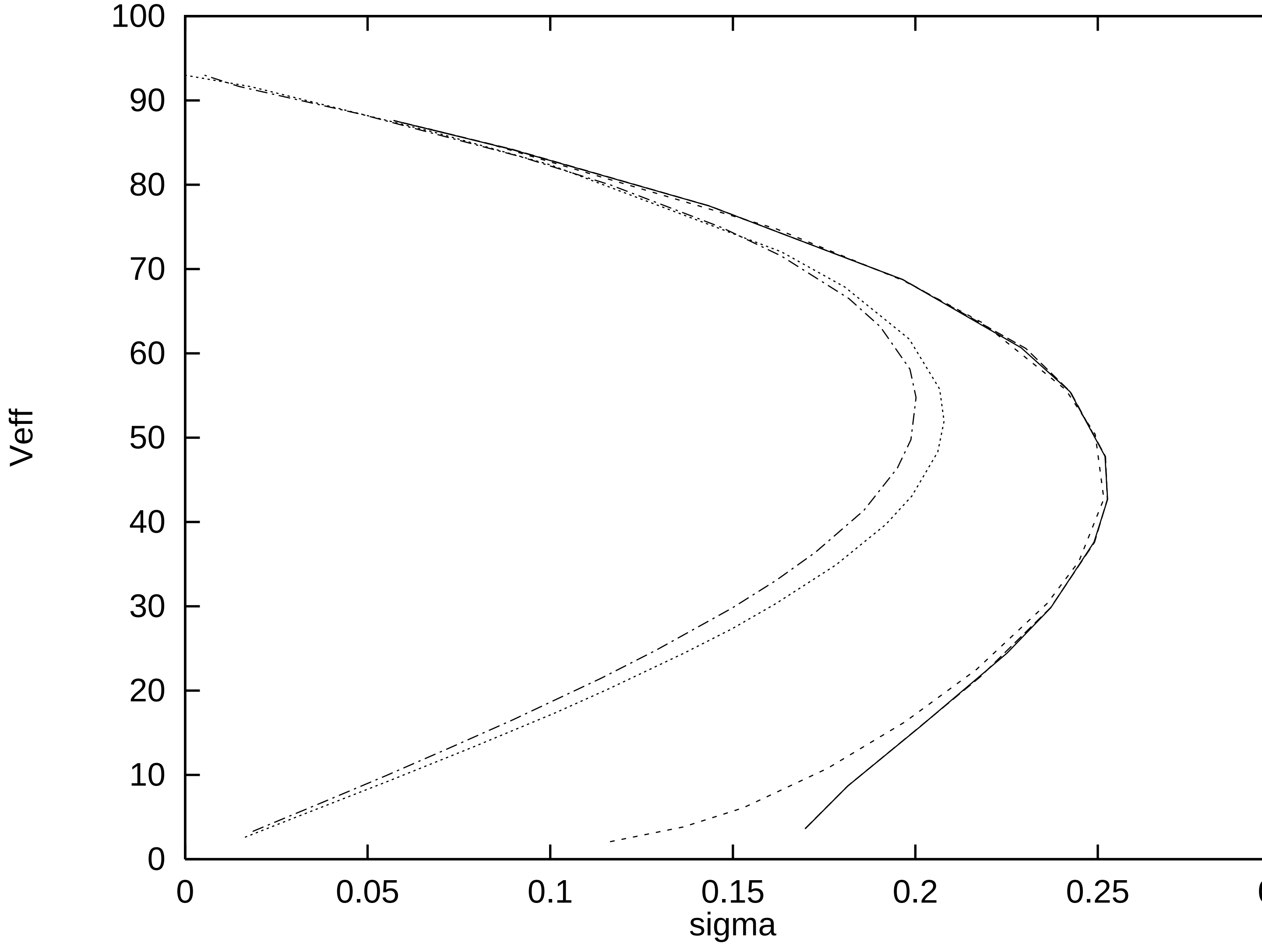

100
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Veff
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0.2
0.25
sigma